INVESTIGATION OF EXTENSIONS TO THE DISTORTED BORN APPROXIMATION IN STRONG FLUCTUATION THEORY

FINAL REPORT

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OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
ARLINGTON, VIRGINIA 22217

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by

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INTRODUCTION AND SUMMARY

This study, funded under Contract N00014-87-0784, considers some scattering problems relevant to the analysis of brightness temperatures and scattering cross sections of sea ice and its snow cover. It is a continuation of work performed at Aerojet ElectroSystems Company under previous contracts with the Office of Naval Research. The theme of these studies has been the application of strong fluctuation theory to the description of the microwave radiometric properties of random media such as sea ice and snow.

These past studies have demonstrated the applicability of the bilocal approximation to the equations of strong fluctuation theory in calculating electromagnetic material properties of random media such as effective dielectric constants [1], [2]. The resulting equations have received confirmation from numerous experiments (see [3], [4] for measurements on snow and the discussion in [2] for experimental data on sea ice). To the present time, these are the only published equations accounting for the order of magnitude increase in the dielectric loss experienced by snow at frequencies above 20 GHz as compared to its low frequency value.

The bilocal approximation has also been used in conjunction with the distorted Born approximation in calculating the brightness temperature of snow [3] and sea ice [5]. The distorted Born approximation, which uses only the mean electric field within the random medium to evaluate scattering and thermal emission into the air above, has had some success in

explaining measurements of the brightness temperature of snow at frequencies below 37 GHz, young and first year ice at frequencies up to at least 140 GHz, and old sea ice at frequencies into the 20 GHz range. Its use in conjunction with strong fluctuation theory has led to results considerably superior to that found by the application of radiative transfer theory. However, for media such as snow and old sea ice, the distorted Born approximation leads to predictions of emissivities and brightness temperature which are considerably higher than observed values at sufficiently high frequencies. The reason for this behavior is suspected to lie in the neglect of scattering by the random, or incoherent, field within the medium when calculating the emissivity and scattering coefficients.

It is the purpose of the work reported here to examine how ~~this~~ heretofore neglected contribution of the incoherent field to the emissivity and scattering cross sections of a random medium may be taken into account. ~~There are~~ two main technical problems which are solved. In Section II, strong fluctuation theory equations are derived for the second moments of the electric field in an anisotropic medium. Previous work in this area had not considered anisotropy in the random medium so that it was not clear what form the second moment equations would take in the case of a general random medium such as sea ice, where anisotropy may occur. Nor was the most useful explicit equation written for the isotropic case. The analysis is dependent on obtaining equations which allow the elimination of the auxiliary field $\underline{F}$, which always arises in strong fluctuation theory, in favor of the electric field $\underline{E}$, which is easy to work with when electromagnetic

boundary conditions are applied and which has a more direct physical meaning than $\underset{\sim}{F}$.

Section III of this report makes use of a particular equation derived in Section II as the basis for a detailed reduction of the second moment equation to a form which is practical for numerical computation. The problem of a layered medium with a spherically symmatric correlation function is considered. The spherical symmetry of the correlation function implies that a scalar description of the dielectric properties of the medium is valid and results in the elimination of several technical problems which would arise had an anisotropic correlation function been assumed. Thus, the theory is applicable to snow cover on sea ice as well as old sea ice (provided that the correlation functions describing brine pockets in the ice is spherically symmetric as is suggested by some studies) but will not be directly applicable to young and first year ice which are anisotropic. However, the development of the theory is general enough to allow complex vertical variations in the mean dielectric profile of the medium so that it may be applied to realistic layered structures found in naturally occurring snow and ice. Particular care is taken to identify and treat all singular behavior which could cause problems in a numerical solution.

The equations developed in Section III have not yet been programmed for solution on a digital computer. In view of the expectation, based on good physical arguments, that the use of these equations will greatly

extend the high frequency range of validity of strong fluctuation theory over the distorted Born approximation for strongly scattering media, it is recommended that the next phase of these studies be devoted to writing a FORTRAN program to solve these equations. There also exists the possibility that one of the integrations involved in solving these equations may be performed analytically, thus substantially decreasing the complexity of the proposed computer program. Hence, an additional analytic effort in this area would be well worthwhile. It is further recommended that the computer program that is written be applied to examining the brightness temperature behavior of an isothermal, dry snow pack at microwave frequencies where the distorted Born approximation begins to fail. Comparisons with experimental data should be made. There is sufficient experimental data available to make such comparisons enlightening. It should also be useful to see what these new equations predict for the brightness temperature of old sea ice (at a uniform temperature) assuming that the applicable brine pocket correlation function is spherically symmetric. The effect of snow cover, which usually occurs on the ice, should also be studied.

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STRONG FLUCTUATION THEORY EQUATIONS FOR ELECTRIC FIELD SECOND MOMENTS IN ANISOTROPIC MEDIA

I. INTRODUCTION

Strong fluctuation theory for random media ([1]-[5]) has found application to numerous interesting geophysical problems. In [6], it was shown how the equations for the mean electric field in the bilocal approximation and the random field in the distorted Born approximation could be formulated directly in terms of the electric field instead of in terms of equations containing an auxiliary field $\underset{\sim}{F}$ (see [1]-[4], [6]). Brekhovskikh [7] has shown how equations for electric field second moments could also be obtained in strong fluctuation theory in the case where the fluctuations in the random medium have a spherically symmetric correlation function and hence lead to a scalar rather than a second rank tensor description of the quantity ξ([1]-[6]) which is used to describe the fluctuations of the medium in strong fluctuation theory. In this communication, the equations for the second moments are generalized to the case of an anisotropic medium.

II. SECOND MOMENT EQUATIONS

The electric field $\underset{\sim}{E}(\underset{\sim}{r})$ in a random medium described by a random second rank dielectric tensor K^r is governed by the equation

$$(L_o+\Delta L)\,\underset{\sim}{E} = 0 \tag{1}$$

where $L_o+\Delta L$ is the operator $\nabla X\, \nabla X - k_o{}^2 K^r$ and k_o is the free space

propagation constant. In strong fluctuation theory a non-random quasi-static dielectric tensor K_o is introduced in terms of which the operators L_o and ΔL are given by

$$(L_o)_{ij} = \frac{\partial^2}{\partial x_i \partial x_j} - \delta_{ij}\nabla^2 - k_o^2 (K_o)_{ij} \tag{2}$$

$$\Delta L = k_o^2 (K_o - K^r) \tag{3}$$

The quasi-static dielectric tensor is defined by means of the equation

$$< \xi > = 0 \tag{4}$$

where the angular brackets denote expected value and

$$\xi = \Delta L[1+S\Delta L]^{-1} \tag{5}$$

Here, all second rank tensors are represented by matrices and multiplication indicates matrix multiplication. The matrix S in (5) is defined as the coefficient of the delta function in the Green's function Γ determined by (2) which may be expressed as

$$\Gamma(\underset{\sim}{r}, \underset{\sim}{r}') = S\,\delta(\underset{\sim}{r}-\underset{\sim}{r}') + PV\,\Gamma'(\underset{\sim}{r},\underset{\sim}{r}') \tag{6}$$

In this equation, PV means that the principal value is to be taken in

integrals in which Γ' appears. With these definitions, an auxiliary field

$$\underset{\sim}{F} = [1+S\Delta L]\, \underset{\sim}{E} \tag{7}$$

is introduced in strong fluctuation theory. It has the property that it satisfies an integral equation whose kernel is free of delta functions, namely

$$\underset{\sim}{F} = G\,\xi\, \underset{\sim}{F} \tag{8}$$

where G is the operator whose kernel is $PV\Gamma'$. Equations(5) and (7) show that

$$\xi\, \underset{\sim}{F} = \Delta L \underset{\sim}{E} \tag{9}$$

while (7) and (9) show that

$$\underset{\sim}{F} = \underset{\sim}{E} + S\,\xi\, \underset{\sim}{F} \tag{10}$$

or

$$\underset{\sim}{E} = [1- S\,\xi\,]\, \underset{\sim}{F} \tag{11}$$

In the following derivation of the second moment equations, the arguments of the various quantities involved will be abbreviated to subscripts. Thus, e.g., the notation

$$\langle \underset{\sim}{E}_1\, \underset{\sim}{E}_2^* \rangle = \langle \underset{\sim}{E}\,(\underset{\sim}{r}_1)\ \underset{\sim}{E}^*(\underset{\sim}{r}_2)\rangle \tag{12}$$

is introduced where* indicates complex conjugate and the juxtaposition of two vectors is simply the matrix representing the direct product of the vectors. Following [7], it is convenient to define the operators ζ and B

by the equations

$$\zeta_1 \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = \langle \xi_1 \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle \quad (13)$$

$$\zeta_2 \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = \langle \underset{\sim}{F}_1 \xi_2^* \underset{\sim}{F}_2^* \rangle \quad (14)$$

$$\zeta_3 \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = \langle \underset{\sim}{F}_1 S_2^* \xi_2^* \underset{\sim}{F}_2^* \rangle \quad (15)$$

$$B \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = \langle \xi_1 \underset{\sim}{F}_1 \xi_2^* \underset{\sim}{F}_2^* \rangle \quad (16)$$

$$B_1 \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = \langle S_1 \xi_1 \underset{\sim}{F}_1 \xi_2^* \underset{\sim}{F}^* \rangle \quad (17)$$

$$B_2 \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = \langle \xi_1 \underset{\sim}{F}_1 S_2^* \xi_2^* \underset{\sim}{F}_2^* \rangle \quad (18)$$

Equations (15), (17), and (18) are additional definitions that are required compared with [7] because S does not commute with the fields in general. For the special case of a spherically symmetric correlation function for ξ and scalar K^r (and hence scalar K_o), S is simply the scalar [8]

$$S = -1/(3\, k_o^2 K_o) \quad (19)$$

and may be removed from the angular brackets in these equations.

The operators ζ and B allow a relation between $\langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle$ and $\langle \underset{\sim}{E}_1 \underset{\sim}{E}_2^* \rangle$

to be established. A calculation using (7), (9) and (11) yields

$$\underset{\sim}{F}_1\,\underset{\sim}{F}_2^{\,*} = \underset{\sim}{E}_1\,\underset{\sim}{E}_2^{\,*} + [1 - S_1\,\xi_1]\,\underset{\sim}{F}_1\,S_2^{\,*}\,\xi_2^{\,*}\,\underset{\sim}{F}_2^{\,*} + S_1\,\xi_1\,\underset{\sim}{F}_1\,[1 - S_2^{\,*}\,\xi_2^{\,*}]\underset{\sim}{F}_2^{\,*}$$

$$+ S_1\,\underset{\sim}{F}_1\,S_2^{\,*}\,\xi_2^{\,*}\,\underset{\sim}{F}_2^{\,*} \tag{20}$$

which has the consequence

$$\langle\underset{\sim}{F}_1\,\underset{\sim}{F}_2^{\,*}\rangle = [1 - \zeta_3 - S_1\,\zeta_1 + S_1\,B_2]^{-1}\,\langle\underset{\sim}{E}_1\,\underset{\sim}{E}_2^{\,*}\rangle \tag{21}$$

Now (9) and (1) show that

$$\xi_1\,\underset{\sim}{F}_1\,\xi_2^{\,*}\,\underset{\sim}{F}_2^{\,*} = \Delta L\underset{\sim}{E}_1\,\Delta L_2^{\,*}\,\underset{\sim}{E}_2^{\,*}$$

$$= L_{o1}\,\underset{\sim}{E}_1\,L_{o2}^{\,*}\,\underset{\sim}{E}_2^{\,*} \tag{22}$$

so that, using (16) and (21), the equation

$$\left\{L_{o1}\,\hat{L}_{o2}^{\,*} - B[1 - \zeta_3 - S_1\,\zeta_1 + S_1\,B_2]^{-1}\right\}\langle\underset{\sim}{E}_1\,\underset{\sim}{E}_2^{\,*}\rangle = 0 \tag{23}$$

is found. Here $\hat{L}_{o2}^{\,*}$ is defined as

$$(\hat{L}_{o2}^{\,*})_{ijk\ell}\,\langle\underset{\sim}{E}_1\,\underset{\sim}{E}_2^{\,*}\rangle_{k\ell} = \delta_{ik}\,(L_{o2}^{\,*})_{j\ell}\,\langle\underset{\sim}{E}_1\,\underset{\sim}{E}_2^{\,*}\rangle_{k\ell} \tag{24}$$

where the summation convention is used for repeated subscripts. Another interesting equation is obtained from the pair of equations

$$\xi_1 \underset{\sim}{F}_1 S_2^* \xi_2^* \underset{\sim}{F}_2^* = \Delta L_1 \underset{\sim}{E}_1 S_2^* \Delta L_2^* \underset{\sim}{E}_2^* \tag{25}$$

$$\xi_1 \underset{\sim}{F}_1 \underset{\sim}{F}_2^* = \Delta L_1 \underset{\sim}{E}_1 \underset{\sim}{F}_2^* = \Delta L_1 \underset{\sim}{E}_1 [\underset{\sim}{E}_2^* + S_2^* \Delta L_2^* \underset{\sim}{E}_2^*] \tag{26}$$

which follows from (17) and (9). Subtracting (25) from (26), using (1), and taking expected values yields

$$L_{o1} \langle \underset{\sim}{E}_1 \underset{\sim}{E}_2^* \rangle + [\zeta_1 - B_2] \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = 0 \tag{27}$$

Of course, the substitution of (21) in (27) yields an equation containing $\langle \underset{\sim}{E}_1 \underset{\sim}{E}_2^* \rangle$ alone. The equations (again following from (7) and (9))

$$S_1 \xi_1 \underset{\sim}{F}_1 \xi_2^* \underset{\sim}{F}_2^* = S_1 \Delta L_1 \underset{\sim}{E}_1 \Delta L_2^* \underset{\sim}{E}_2^* \tag{28}$$

$$\underset{\sim}{F}_1 \xi_2^* \underset{\sim}{F}_2^* = \underset{\sim}{F}_1 \Delta L_2^* \underset{\sim}{E}_2^* = [\underset{\sim}{E}_1 + S_1 \Delta L_1 \underset{\sim}{E}_1] \Delta L_2^* \underset{\sim}{E}_2^* \tag{29}$$

yield an equation analogous to (27):

$$\hat{L}_{o2}^* \langle \underset{\sim}{E}_1 \underset{\sim}{E}_2^* \rangle + [\zeta_2 - B_1] \langle \underset{\sim}{F}_1 \underset{\sim}{F}_2^* \rangle = 0 \tag{30}$$

III. APPROXIMATIONS

Equations (23), (27), and (30) are exact. If, as in [6], quadratic

and higher order terms in ζ and B are ignored, then the equations

$$\left\{L_{o1}\ \hat{L}_{o2}{}^{*} - B\right\} \langle \underset{\sim}{E}_1\ \underset{\sim}{E}_2{}^{*}\rangle = 0 \tag{31}$$

$$\left\{L_{o1} + \zeta_1 - B_2\right\} \langle E_1\ E_2{}^{*}\rangle = 0 \tag{32}$$

$$\left\{\hat{L}_{o2}{}^{*} + \zeta_2 - B_1\right\} \langle \underset{\sim}{E}_1\ \underset{\sim}{E}_2{}^{*}\rangle = 0 \tag{33}$$

are found. Allowing for slight differences in notation, eq (32) reduces to that given in [7] (written for a vanishing external current density) if the medium is isotropic so that (19) may be used.

A slightly different version of (31) is useful in problems where the mean field $\langle \underset{\sim}{E}\rangle$ has already been calculated and an expression for the incoherent contribution to $\langle \underset{\sim}{E}_1\ \underset{\sim}{E}_2{}^{*}\rangle$ is needed. That is, if the field is written as

$$\underset{\sim}{E} = \langle \underset{\sim}{E}\rangle + \underset{\sim}{E}^{r} \tag{34}$$

where $\underset{\sim}{E}^{r}$ is the random part of $\underset{\sim}{E}$, then $\langle \underset{\sim}{E}_1{}^{r}\ \underset{\sim}{E}_2{}^{r*}\rangle$ is to be found. Perhaps the easiest way to derive the desired equation is to note that, according to [6], $\underset{\sim}{E}^{r}$ satisfies

$$\underset{\sim}{E}^{r} = -L_o{}^{-1}\left\{\xi \underset{\sim}{F} - \langle \xi \underset{\sim}{F}\rangle\right\} \tag{35}$$

exactly. Then

$$<\underset{\sim}{E}_1{}^r\ \underset{\sim}{E}_2{}^{r*}> = L_{o1}{}^{-1}\ \hat{L}^*_{o2}{}^{-1} \left\{ B < \underset{\sim}{F}_1\ \underset{\sim}{F}_2{}^*> - <\xi_1\ \underset{\sim}{F}_1> <\xi_2{}^*\ \underset{\sim}{F}_2{}^*> \right\} \tag{36}$$

Now, for large separations $\underset{\sim}{r}_1-\underset{\sim}{r}_2$, (16) shows that $B <\underset{\sim}{F}_1\ \underset{\sim}{F}_2{}^*> \longrightarrow$ $<\xi_1\ \underset{\sim}{F}_1> <\xi_2{}^*\ \underset{\sim}{F}_2{}^*>$ so that the bracketed quantity in (36) vanishes. On the other hand, for small to moderate values of $\underset{\sim}{r}_1-\underset{\sim}{r}_2$, the first term in the bracket dominates. Thus, if it is assumed that the field at a point is only slightly dependent on the fluctuation at that point, (16) yields

$$\left\{ B <\underset{\sim}{F}_1\ \underset{\sim}{F}_2{}^*> \right\}_{ij} \approx (C'_{12})_{ijk\ell} <F_1 F_2{}^*>_{k\ell} \tag{37}$$

where

$$(C'_{12})_{ijk\ell} = < (\xi_1)_{ik}\ (\xi_2{}^*)_{j\ell}> \tag{38}$$

is the 4th rank covariance tensor for the random tensors ξ and ξ^*. Since (38) vanishes for large $\underset{\sim}{r}_1-\underset{\sim}{r}_2$, a reasonable approximation to the bracket in (36) is simply (37). Further, since higher order terms are already ignored in (31), $<\underset{\sim}{F}_1\ \underset{\sim}{F}_2{}^*>$ in (37) may be replaced by $<\underset{\sim}{E}_1\ \underset{\sim}{E}_2{}^*>$. Thus

$$<\underset{\sim}{E}_1{}^r\ \underset{\sim}{E}_2{}^{r*}> = L_{o1}{}^{-1}\ \hat{L}^*_{o2}{}^{-1}\ C'_{12} <\underset{\sim}{E}_1\ \underset{\sim}{E}_2{}^*> \tag{39}$$

Upon noting that (34) yields

$$<\underset{\sim}{E}_1\ \underset{\sim}{E}_2{}^*> = <\underset{\sim}{E}>_1 <\underset{\sim}{E}>_2 + <\underset{\sim}{E}_1{}^r \underset{\sim}{E}_2{}^{r*}> \tag{40}$$

(35) may be replaced by

$$\left\{ L_{01} \hat{L}_{o2}{}^{*} - C_{12}' \right\} < \underset{\sim}{E}_1{}^{r} \underset{\sim}{E}_2{}^{r*} > = C_{12}' < \underset{\sim}{E} >_1 < \underset{\sim}{E}^{*} >_2 \tag{41}$$

Either (39) or (41) represent a suitable formulation for practical computation.

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SECTION 2

STRONG FLUCTUATION THEORY CALCULATION OF SCATTERED FIELDS FROM MULTI-LAYERED RANDOM MEDIA

STRONG FLUCTUATION THEORY CALCULATION OF SCATTERED FIELDS FROM MULTI-LAYERED RANDOM MEDIA

I. INTRODUCTION

The theory of electromagnetic scattering and microwave thermal emission from random media such as snow, sea ice, and vegetation has become an important area of research. One approach to studying this problem has been strong fluctuation theory [1], [2] which has been applied to calculating effective dielectric constants [3] of snow and sea ice [4]-[6] (with strong experimental support [7]), scattering cross sections of vegetation [8], [9] and snow [10], and the microwave brightness temperature of dry and wet snow [3]. In applications to scattering and brightness temperature calculations, the distorted Born approximation has been used in the past. This approximation neglects scattering of the incoherent field and results in an underestimate of the true scattering by the medium. Its effect is particularly noticeable at higher frequencies.

It is the purpose of this work to improve upon the distorted Born approximation in strong fluctuation theory. Integral equations are developed for the second moments of the incoherent electric field in the case of a multi-layered random medium, each of whose layers is bounded by plane surfaces. It is shown how the solution of these equations is incorporated into the expression for the bistatic scattering cross section of the medium. Of course, by use of the results of Peake [11], these scattering cross sections may be used to calculate the thermal emissivity of the medium if it is at a uniform temperature.

II. BISTATIC SCATTERING COEFFICIENTS

Consider a layered medium, as is illustrated in Fig. 1, that has a random scalar dielectric constant $K^r(\underset{\sim}{r})$ dependent on the point $\underset{\sim}{r}$ for $z<0$. It is assumed that the expected value of K^r is a function of z only and that the covariance function of K^r is spherically symmetric (i.e. a function of the magnitude of the separation of two points only). Further, assume that changes in the expected value of K^r over a correlation length are small (except, of course, at possible boundaries where discontinuities of the dielectric properties arise).

Suppose that the medium is illuminated by an incident plane wave

$$\underset{\sim}{E}_{in} = E_o \hat{\underset{\sim}{p}}_o \exp[i\underset{\sim}{k}_o \cdot \underset{\sim}{r}] \tag{1}$$

where

$$\underset{\sim}{k}_o = k_o(\sin\theta_o, 0, -\cos\theta_o) \tag{2}$$

and $\hat{\underset{\sim}{p}}$ is a unit polarization vector which will be taken to be either horizontal (h) or vertical (v):

$$\hat{p}_o = \begin{cases} \hat{\underset{\sim}{h}}_o = (0,-1,0) \\ \text{or} \\ \hat{\underset{\sim}{v}}_o = (\cos\theta_o, 0, \sin\theta_o) \end{cases} \tag{3}$$

We are interested in the scattered power contained in a small solid single about an arbitrary propagation vector

$$\underset{\sim}{k} = k_o(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \tag{4}$$

The electric field which is generated by the wave (1) may be expressed as

$$\underset{\sim}{E} = \underset{\sim}{E}^m + \underset{\sim}{E}^r \tag{5}$$

where $\underset{\sim}{E}^m$ is the mean wave and $\underset{\sim}{E}^r$ the random, or incoherent part. Using the bilocal approximation of strong fluctuation theory, $\underset{\sim}{E}^m$ satisfies the equation [12]

$$\left\{L_o - \langle \xi G \xi \rangle\right\} \underset{\sim}{E}^m = 0 \tag{6}$$

where L_o is the operator

$$L_o = \begin{cases} \nabla x \nabla x - k_o^2 & \text{if } z>0 \\ \nabla x \nabla x - k_o^2 K_o & \text{if } z<0 \end{cases} \tag{7}$$

and G is the non-delta function part of the dyadic Green's function $\underset{\approx}{\Gamma}(\underset{\sim}{r}, \underset{\sim}{r}')$ for (7). Here, angular brackets are used to denote expected values and K_o is the quasistatic dielectric constant of the medium and is defined by the

equation

$$\langle\xi\rangle = 0 \tag{8}$$

where

$$\xi = \Delta L[1+S\Delta L]^{-1} \tag{9}$$

and

$$\Delta L = k_o^{\,2}(K_o - K^r) \tag{10}$$

In (9), $S = -1/(3k_o^{\,2}K_o)$ is the coefficient of the delta function part of $\underset{\approx}{\Gamma}$. The second moment dyad of the incoherent field satisfies the equation [13]

$$\langle \underset{\sim}{E}^r(r_1)\underset{\sim}{E}^{r*}(r_2)\rangle = \int d^3r' d^3r'' C'(|\underset{\sim}{r}'-\underset{\sim}{r}''|)\underset{\approx}{\Gamma}(\underset{\sim}{r}_1,\underset{\sim}{r}')\langle\underset{\sim}{E}(r')\underset{\sim}{E}^*(r'')\rangle\underset{\approx}{\Gamma}^{\dagger}(\underset{\sim}{r}_2,\underset{\sim}{r}'') \tag{11}$$

where

$$C'(|\underset{\sim}{r}'-\underset{\sim}{r}''|) = \langle\xi(\underset{\sim}{r}')\xi^*(\underset{\sim}{r}'')\rangle \tag{12}$$

is spherically symmetric according to the assumptions made above. Here * indicates complex conjugate, and † indicates adjoint. Notice that (5) implies that

$$\langle\underset{\sim}{E}(\underset{\sim}{r}_1)\underset{\sim}{E}(\underset{\sim}{r}_2) = \underset{\sim}{E}^m(\underset{\sim}{r}_1)\underset{\sim}{E}^{m*}(\underset{\sim}{r}_2) + \langle\underset{\sim}{E}^r(\underset{\sim}{r}_1)\underset{\sim}{E}^{r*}(\underset{\sim}{r}_2)\rangle \tag{13}$$

When this is used in the right hand side of (11), it is seen that (11) is an integral equation for $\langle\underset{\sim}{E}^r(\underset{\sim}{r}_1)\underset{\sim}{E}^{r*}(\underset{\sim}{r}_2)\rangle$.

The bistatic scattering coefficients defined in [11] require the decomposition of the left hand side of (11), evaluated at $\underset{\sim}{r}_1 = \underset{\sim}{r}_2 = \underset{\sim}{r}$, into an

angular spectrum. Namely, the components of (11) are written as

$$<E_i^r(\underset{\sim}{r})E_j^{r*}(\underset{\sim}{r})>=\int d\Omega <E_i^r(\underset{\sim}{k})E_j^{r*}(\underset{\sim}{k})> \tag{14}$$

where $d\Omega$ is an element of solid angle about the vector $\underset{\sim}{k}$ and the integral is taken over the upper hemisphere. In (14), $\underset{\sim}{E}(\underset{\sim}{k})$ denotes the Fourier transform of $\underset{\sim}{E}(\underset{\sim}{r})$. The scattering coefficients are

$$\gamma_{ab}(\underset{\sim}{k}_o,\underset{\sim}{k})= (4\pi\cos\theta/E_o^2\cos\theta_o)b_i b_j <E_i^r(\underset{\sim}{k})E_j^{r*}(\underset{\sim}{k})> \tag{15}$$

where $\hat{\underset{\sim}{a}}$ and $\hat{\underset{\sim}{b}}$ denote unit vectors describing the polarization of the incident and scattered waves respectively. We will be concerned with the case where $\hat{a}$ is given by (3) and $\hat{\underset{\sim}{b}}$ represents horizontal or vertical polarization for the propagation vector (4):

$$\hat{\underset{\sim}{b}} = \begin{cases} \hat{\underset{\sim}{h}} = (\sin\phi,-\cos\phi,0) \\ \text{or} \\ \hat{\underset{\sim}{v}} = (-\cos\theta\cos\phi,-\cos\theta\sin\phi,\sin\theta) \end{cases} \tag{16}$$

The computation of $\underset{\sim}{E}^m$ for a layered medium of the type being considered here has been considered in detail previously [5], [14]. These works also provide detailed expressions for $<\underset{\sim}{E}^r(\underset{\sim}{k})\underset{\sim}{E}^{r*}(\underset{\sim}{k})>$ in the distorted Born approximation which is obtained by neglecting the term $<\underset{\sim}{E}^r(\underset{\sim}{r}')\underset{\sim}{E}^{r*}(\underset{\sim}{r}'')>$ when (13) is substituted into the right hand side of (11). The task here is to

retain this term in (11). Thus, write (15) as

$$\gamma_{ab}(\underset{\sim}{k}_o, \underset{\sim}{k}) = \gamma^B_{ab}(\underset{\sim}{k}_o, \underset{\sim}{k}) + \gamma^I_{ab}(\underset{\sim}{k}_o, \underset{\sim}{k}) \qquad (17)$$

where γ^B_{ab} represents the distorted Born approximation contribution to γ_{ab} and may be considered to be known. The contribution of the incoherent field γ^I_{ab} arises from the second term on the right hand side of (13) when substituted into the right hand side of (11) and yields, using the notation for the two dimensional Fourier transform of the Green's function developed in [14],

$$\gamma^I_{ab}(\underset{\sim}{k}_o, \underset{\sim}{k}) = [k_o^6 \cos\theta/(\pi\cos\theta_o)] b_i b_j \int_{-\infty}^{0} dz' \int_{-\infty}^{0} dz'' A_{ik}(z', k_x, k_y)$$
$$A^*_{j\ell}(z'', k_x, k_y) F_{k\ell}(k_y, k_y, z', z'') \qquad (18)$$

where the dyad $\underset{\approx}{F}$ is the two dimensional Fourier transform

$$\underset{\approx}{F}(k_y, k_y, z', z'') = \int_{-\infty}^{\infty} dxdy\; C'(|r'-r''|)\exp[ik_x x + ik_y y] \langle \underset{\sim}{E}^r(\underset{\sim}{r}')\underset{\sim}{E}^{r*}(\underset{\sim}{r}'')\rangle \qquad (19)$$

and the summation convention is used for repeated Latin letter subscripts. In (19), in addition to the z', z'' dependence, the expected value on the right hand side is a function of $x = x'-x''$ and $y = y'-y''$ because of the horizontal translational invariance of the problem under consideration.

Considerable simplification in further calculations may be achieved if use is made of an additional symmetry of the problem. Notice that the driving term in (11) is $\underset{\sim}{E}^m\underset{\sim}{E}^{m*}$ which, in turn, is determined by the incident wave term $\underset{\sim}{E}_{in}(\underset{\sim}{r}_1)\underset{\sim}{E}^*_{in}(\underset{\sim}{r}_2)$ arising from (2). But, for either horizontal or vertical polarization, this dyad is invariant under reflections in the x, z

plane. The geometry illustrated in Fig. 1 as well as the expected value of the fluctuations is also invariant under this reflection. Hence, so is $\langle \underset{\sim}{E}^r(\underset{\sim}{r}_1)\underset{\sim}{E}^{r*}(\underset{\sim}{r}_2)\rangle$. But this implies that the (x,y), (y,x), (y,z) and (z,y) components vanish. Hence, we may write, in matrix notation,

$$\langle \underset{\sim}{E}^r(\underset{\sim}{r}_1)\underset{\sim}{E}^{r*}(\underset{\sim}{r}_2)\rangle = \begin{pmatrix} e_{11} & 0 & e_{13} \\ 0 & e_{22} & 0 \\ e_{31} & 0 & e_{33} \end{pmatrix} \tag{20}$$

From (19), an identical symmetry holds for $\underset{\approx}{F}$.

With this information, a routine calculation following that described in [14] shows that, for $\hat{\underset{\sim}{b}} = \hat{\underset{\sim}{h}}$ (see (16)), (18) reduces to

$$\gamma^I_{ah} = k_o^6 \cos^2\theta/(\pi\cos\theta_o)\int_{-\infty}^{0} dz' \int_{-\infty}^{0} dz'' A_{\phi\phi}(z',\theta)A^*_{\phi\phi}(z'',\theta)\cdot$$

$$[\sin^2\phi\, F_{11}(z',z'')+\cos^2\phi\, F_{22}(z',z'')] \tag{21}$$

while for $\hat{\underset{\sim}{b}} = \hat{\underset{\sim}{v}}$

$$\gamma^I_{av} = k_o^6/(\pi\cos\theta_o)\int_{-\infty}^{0} dz' \int_{-\infty}^{0} dz'' \Big\{$$

$$A_{\rho\rho}(z',\theta)A^*_{\rho\rho}(z'',\theta)[\cos^2\phi\, F_{11}(z',z'')+\sin^2\phi\, F_{22}(z',z'')] \tag{22}$$

$$+\cos\phi[A_{\rho\rho}(z',\theta)A^*_{\rho z}(z'',\theta)F_{13}(z',z'')+A_{\rho z}(z',\theta)\,A^*_{\rho\rho}(z'',\theta)F_{31}(z',z'')]$$

$$+\quad A_{\rho z}(z',\theta)A^*_{\rho z}(z'',\theta)F_{33}(z',z'')\Big\}$$

The k_o dependence of all quantities in the integrands of (21) and (22) has been suppressed for simplicity. Eqs (21) and (22), in conjunction with the known γ_{ab}^{B}, provide a complete description of the scattering coefficients (17). It is interesting to observe that (21) and (22) yield non-vanishing cross polarized returns in the special case of backscattering $\theta = \theta_o, \phi = \pi$. Of course, the distorted Born approximation does not lead to cross-polarized backscatter cross sections.

An approximation which is very useful in simplifying later calculations will be introduced here. For a sharply peaked correlation function C' in (19), the expected value may be removed from the integrand to yield

$$F_{k\ell}(k_x, k_y, z', z'') \approx E_{k\ell}(z') W'(z', z'', k_x, k_y) \tag{23}$$

where, from (20),

$$E_{k\ell}(z') = e_{k\ell}(\underset{\sim}{r}', \underset{\sim}{r}') \tag{24}$$

and

$$W'(z', z'', k_x, k_y) = \int_{-\infty}^{\infty} dxdy\; C'(|x|, |y|, |z-z'|) \exp[ik_x x + ik_y y] \tag{25}$$

Notice that translational invariance in the horizontal direction implies that $E_{k\ell}$ does not depend on x' or y'.

Equations (21)-(25) define the incoherent wave contribution to the scattering cross sections. The quantities $A_{\phi\phi}$, $A_{\rho\rho}$, etc. have been discussed in [5] and [14]. However $E_{k\ell}(z')$ must still be calculated $z'<0$. This will be discussed in the next section.

III INTEGRAL EQUATIONS FOR THE INCOHERENT FIELD SECOND MOMENT

The second moment dyad is governed by (11) where the driving term is the (assumed) known quantity $\underset{\sim}{E}^{m}(\underset{\sim}{r}_1)\underset{\sim}{E}^{m*}(\underset{\sim}{r}_2)$ exhibited in (13). This equation will be reduced to a form which is more suitable for computational work. The principal tool in this reduction is the representation and properties of the Green's dyad discussed in Appendix A.

When eq. (A-1) is substituted into (11), several delta functions arise which can immediately be used to reduce the number of integrals to be performed. It is found that

$$\langle \underset{\sim}{E}^{r}(\underset{\sim}{r}_1)\underset{\sim}{E}^{r*}(\underset{\sim}{r}_2)\rangle = (2\pi)^{-2}\int_{\infty}^{0} dz'\, dz''\int_{\infty}^{\infty} d\xi d\eta\, \underset{\approx}{A}'(z_1,z',\xi,\eta)\underset{\approx}{F}(\xi,\eta,z',z'')\underset{\approx}{A}'(z_2,z'',\xi,\eta)$$

$$\exp[i\xi(x_1-x_2)+i\eta(y_1-y_2)] \qquad (26)$$

where $\underset{\approx}{F}$ is defined in (19). If the correlation function C' is assumed to be sharply peaked, then eqs. (23)-(25) will be applicable for use in computing γ_{ab}^{I}. But this implies that the solution of (26) will be of interest only for $\underset{\sim}{r}_1 = \underset{\sim}{r}_2 = \underset{\sim}{r}$. Thus

$$\underset{\approx}{E}(z) = (2\pi)^{-2}\int dz'\, dz'' d\xi d\eta\, W'(z',z'',\xi,\eta)\underset{\approx}{A}'(z,z',\xi,\eta)\langle \underset{\sim\sim}{EE}^{*}\rangle(z')\underset{\approx}{A}'^{\dagger}(z,z'',\xi,\eta) \qquad (27)$$

where the components of $\underset{\approx}{E}$ are defined by (24) and, according to (13),

$$\langle \underset{\sim\sim}{EE}^{*}\rangle(z') = \underset{\sim}{E}^{m}(\underset{\sim}{r}')\underset{\sim}{E}^{m*}(\underset{\sim}{r}')+\underset{\approx}{E}(z') \qquad (28)$$

Notice that the first term on the right hand side of (28) is actually a function of z' only because $\underset{\approx}{E}^m$ depends on the horizontal coordinates only through a factor of the form $\exp[ik_o \sin\theta_o x']$(see [14]).

A further reduction of (27) arises from the observation that eq (A-2) shows that the angular dependence of $\underset{\approx}{A}'$ occurs as a simple factors of powers of $\cos\phi$ and $\sin\phi$ if cylindrical coordinates (ρ,ϕ) are introduced by means of the equations

$$\xi = \rho\cos\phi$$
$$\eta = \rho\sin\phi \tag{29}$$

Since the correlation function C' was assumed to be spherically symmetric, (25) implies that the ξ,η dependence of $W'(z',z'',\xi,\eta)$ is simply a dependence on the parameter ρ only. Performing the ϕ integration in (28), recalling the symmetry for $\underset{\approx}{E}$ given in (20), and using (A-2) results in the equation

$$\begin{aligned}
E_{11} &= \int_{\infty}^{0} dz'\, dz' \int_{0}^{\infty} \rho d\rho\, W' \left\{ Q_{11}\langle EE^*\rangle_{11} + Q_{12}\langle EE^*\rangle_{22} + Q_{13}\langle EE^*\rangle_{33} \right\} \\
E_{22} &= \int_{-\infty}^{0} dz'\, dz'' \int_{0}^{\infty} \rho d\rho\, W' \left\{ Q_{12}\langle EE^*\rangle_{11} + Q_{11}\langle EE^*\rangle_{22} + Q_{13}\langle EE^*\rangle_{33} \right\} \\
E_{33} &= \int_{-\infty}^{0} dz'\, dz'' \int_{0}^{\infty} \rho d\rho\, W' \left\{ Q_{31}\langle EE^*\rangle_{11} + Q_{31}\langle EE^*\rangle_{22} + Q_{33}\langle EE^*\rangle_{33} \right\} \\
E_{13} &= \int_{\infty}^{0} dz'\, dz'' \int_{0}^{\infty} \rho d\rho\, W' \left\{ P_1 \langle EE^*\rangle_{13} + P_2 \langle EE^*\rangle_{13} \right\} \\
E_{31} &= E^*_{13}
\end{aligned} \tag{30}$$

where

$$Q_{11}(z,z',z'')=(4\pi)^{-1}\left[a_{\phi\phi}(z,z')a^*_{\phi\phi}(z,z'')+a_{\phi\phi}(z,z')a^*_{\phi\phi}(z,z'')\right]-Q_{12}$$

$$Q_{12}(z,z',z'')=(16\pi)^{-1}\left[a_{\rho\rho}(z,z')-a_{\phi\phi}(z,z')\right]\left[a^*_{\rho\rho}(z,z'')-a^*_{\phi\phi}(z,z'')\right]$$

$$Q_{13}(z,z',z'')=(4\pi)^{-1}a_{\rho z}(z,z')a^*_{\rho z}(z,z'')$$

$$Q_{31}(z,z',z'')=(4\pi)^{-1}a_{z\rho}(z,z')a^*_{z\rho}(z,z'') \tag{31}$$

$$Q_{33}(z,z',z'')=(2\pi)^{-1}a_{zz}(z,z')a^*_{zz}(z,z'')$$

$$P_1(z,z',z'')=(4\pi)^{-1}\left[a_{\phi\phi}(z,z')+a_{\rho\rho}(z,z')\right]a^*_{zz}(z,z'')$$

$$P_2(z,z',z'')=(4\pi)^{-1}a_{\rho z}(z,z')a^*_{\rho z}(z,z'')$$

The dependence of all quantities in (31) on the parameter ρ has been suppressed in the notation.

At this point, it is useful to introduce a variable change in the integrals occurring in (30). Notice that, according to our assumptions, W' decreases very rapidly as a function of the magnitude of $z'-z''$ and is nearly zero over a distance beyond a few correlation lengths. On the other hand, the z' and z'' dependence of the remaining terms in the integrands do not

change much over a correlation length. Thus, the change of variables (compare with [5])

$$
\begin{aligned}
z_1 &= z' \\
u &= z'' - z'
\end{aligned}
\tag{32}
$$

will be made. With the above assumptions concerning W', we indicate the argument dependence of W' as $W'(|u|;z_1,\rho)$ where the z_1 dependence is slow so that, over distances of a few correlation lengths ($|u|$ not too large).

$$
W'(|u|;z_1+u,\rho) \approx W'(|u|;z_1,\rho)
\tag{33}
$$

Likewise, $\langle \underset{\sim}{E}\underset{\sim}{E}^* \rangle$ does not change much over a correlation length so that it may be considered to be a function of z_1 alone in the integrands. Following the procedure in [5] and using these assumptions allows (30) to be approximated as

$$
\begin{aligned}
E_{11} &= \int_{-\infty}^{0} dz_1 \left\{ D_{11}\langle EE^*\rangle_{11} + D_{12}\langle EE^*\rangle_{22} + b_{\rho z}\langle EE^*\rangle_{33} \right\} \\
E_{22} &= \int_{-\infty}^{0} dz_1 \left\{ D_{12}\langle EE^*\rangle_{11} + D_{11}\langle EE^*\rangle_{22} + b_{\rho z}\langle EE^*\rangle_{33} \right\} \\
E_{33} &= \int_{-\infty}^{0} dz_1 \left\{ b_{z\rho}\langle EE^*\rangle_{11} + b_{z\rho}\langle EE^*\rangle_{22} + 2b_{zz}\langle EE^*\rangle_{33} \right\} \\
E_{13} &= \int_{-\infty}^{0} dz_1 \left\{ D_{13}\langle EE^*\rangle_{13} + 1/2\, C_{\rho zz\rho}\langle EE^*\rangle_{13}^* \right\}
\end{aligned}
\tag{34}
$$

where

$$D_{11} = 3/4[b_{\rho\rho} + b_{\phi\phi}] + 1/4 b_{\rho\phi}$$

$$D_{12} = 1/4[b_{\rho\rho} + b_{\phi\phi} - b_{\rho\phi}] \tag{35}$$

$$D_{13} = 1/2[C_{\rho\rho zz} + C_{\phi\phi zz}]$$

Here

$$b_{\alpha\beta}(z,z_1) = (2\pi)^{-1} \mathrm{Re}\int_0^{\infty} \rho d\rho \int_{-\infty}^{0} du\, W'(|u|;z_1) a_{\alpha\beta}(z_1) a^*_{\alpha\beta}(z_1+u) \tag{36}$$

where the subscripts $(\alpha\beta)$ are one of the pairs $(\rho\rho)$, $(\phi\phi)$, (zz), (ρz) or $(z\rho)$ and Re means "real part". The first argument, z, in the components of the Green's dyad is suppressed for simplicity . In (35),

$$b_{\rho\phi}(z,z_1) = (2\pi)^{-1} \mathrm{Re}\int_0^{\infty} \rho d\rho \int_{-\infty}^{0} du\; W'\left[a_{\rho\rho}(z_1) a^*_{\phi\phi}(z_1+u) + a_{\phi\phi}(z_1) a^*_{\rho\rho}(z_1+u)\right] \tag{37}$$

Also

$$C_{\rho z z\rho}(z,z_1) = (2\pi)^{-1}\int_0^{\infty} \rho d\rho \int_{-\infty}^{0} du\; W'\left[a_{\rho z}(z_1) a^*_{z\rho}(z_1+u) + a_{\rho z}(z_1) a^*_{z\rho}(z_1+u)\right] \tag{38}$$

and

$$C_{\alpha\alpha zz}(z,z_1) = (2\pi)^{-1}\int_0^{\infty} \rho d\rho \int_{-\infty}^{0} du\; W'\left[a_{\rho z}(z_1) a^*_{zz}(z_1+u) + a_{\alpha\alpha}(z_1+u) a^*_{zz}(z_1)\right] \tag{39}$$

where the subscript α is either ρ or ϕ.

Equations (34 -(39) are a set of integral equations for $\underset{\approx}{E}$ when $\underset{\approx}{E}^m \underset{\approx}{E}^{m*}$ is specified. However, they are not quite satisfactory for numerical studies

for two reasons. First, according to Appendix A, the terms in the kernels containing a_{zz} have an explicit delta function part which should be treated separately in numerical work. Second, the asymptotic expressions for large values of the parameter ρ given in Appendix A show that special steps must be taken in order to avoid numerical difficulties in evaluating the integrals in (36)-(39).

The explicit delta function contribution from terms containing a_{zz} is easy to treat and results in an explicit evaluation of the integrals over z_1 in eqs (34). Details are given in Appendix B. The second problem concerning large values of the parameter ρ is treated by writing all integrals over ρ as

$$\int_0^{\infty} d\rho = \int_0^{\rho_1} d\rho + \int_{\rho_1}^{\infty} d\rho \tag{40}$$

where the limit ρ_1 is chosen large enough so that the components of the Green's dyad show an exponential decrease. A reasonable choice is

$$\rho_1 = k_o \sqrt{\mathrm{Max}(\mathrm{Re}\, K_o)} \tag{41}$$

where the maximum is over the range of z_1 where the medium is random. The asymptotic expressions developed in Appendix A may be used as an approximation in the second integral in (40). This allows an explicit evaluation of the integrals over z_1 in (34) for large ρ when the order of integrations is reversed so that the integral over (ρ_1, ∞) is last. The details are found in Appendix C.

Of course, the solution of eqs (34) requires the specification of the correlation function C' or, equivalently, its transform W'. We will write equations under the assumption that C' is an exponential function (it will be obvious how to modify the following results for a single exponential if a sum of exponentials is assumed instead):

$$C'(|\underset{\sim}{r}-\underset{\sim}{r}_1|) = \langle|\xi|^2\rangle \exp\left[-|\underset{\sim}{r}-\underset{\sim}{r}_{1}|/\ell\right] \tag{42}$$

where $\langle|\xi|^2\rangle$ is defined by (9) and (12) for $r' = r''$. Then $W'(|u|;z)$ is

$$W' = \langle|\xi|^2\rangle V(|u|;\rho) \tag{43}$$

where

$$V = (2\pi/\ell)p^{-3}\left[1+p|u|\right]\exp\left[-p|u|\right] \tag{44}$$

and

$$p = \left[1+\ell^2\rho^2\right]^{\frac{1}{2}}/\ell \tag{45}$$

The slow dependence of W' indicated by its second argument allows the possibility that $\langle|\xi|^2\rangle$ and/or ℓ may be a function of z. With these equations, all remaining integrals over ρ in the interval (ρ_1,∞) may be evaluated completely as is shown in Appendix D.

Upon using the results in Appendices B, C, and D in eqs (34), it is found that the first three equations of the set (34) may be written as

$$E_{11}+m_1\langle EE^*\rangle_{11}+m_2\langle EE^*\rangle_{22}=\int_{\infty}^{0}dz_1\left\{M_{11}\langle EE^*\rangle_{11}+M_{12}\langle EE^*\rangle_{22}+f_{\rho z}\langle EE^*\rangle_{33}\right\}$$

$$m_2\langle EE^*\rangle_{22}+E_{22}+m_1\langle EE^*\rangle_{22}=\int_{\infty}^{0}dz_1\left\{M_{12}\langle EE^*\rangle_{11}+M_{11}\langle EE^*\rangle_{22}+f_{\rho z}\langle EE^*\rangle_{33}\right\} \tag{46}$$

$$E_{33}+m_3\langle EE^*\rangle_{33}=\int_{\infty}^{0}dz_1\left\{f_{z\rho}\langle EE^*\rangle_{11}+f_{z\rho}\langle EE^*\rangle_{22}+M_{33}\langle EE^*\rangle_{33}\right\}$$

where

$$M_{11}(z,z_1)=3/4f_{\rho\rho}+3/4f_{\phi\phi}+1/4f_{\rho\phi}$$

$$M_{12}(z,z_1)=1/4\left[f_{\rho\rho}+f_{\phi\phi}-f_{\rho\phi}\right] \tag{47}$$

$$M_{33}(z,z_1)=2f_{zz}$$

Here the subscripted functions f are identical to the subscripted functions b defined in (36) and (37) except for the fact that the upper limit in the integration over ρ in (36) and (37) is replaced by ρ_1 (see eq (41)) and a_{zz} is replaced by a_{zz}^{cl} (see Appendix B) wherever it occurs. The multiplying factors m_1, m_2, and m_3 are given by

$$m_1=\langle|\xi|^2\rangle\left\{-3F_2(\ell\rho_1)/\left[32k_o^4|K_o(z)|^2\right]-3\ell^4F_3(\ell\rho_1)/32\right.$$
$$\left.+\ell^2F_1(\ell\rho_1)\mathrm{Re}\left[1/K_o(z)\right]/\left[16k_o^2\right]\right\}$$

$$m_2=\langle|\xi|^2\rangle\left\{-F_2(\ell\rho_1)/\left[32K_o^4|K_o(z)|^2\right]-\ell^4F_3(\ell\rho_1)/32\right.$$
$$\left.-\ell^2F_1(\ell\rho_1)\mathrm{Re}\left[1/K_o(z)\right]/\left[16k_o^2\right]\right\} \tag{48}$$

$$m_3 = <|\xi|^2>[\tfrac{11}{4}F_2(\ell\rho_1)-2]/[4k_o^4|K(z)|^2]+\int_0^{\rho_1}\rho d\rho \, Re\left\{\int_{-\infty}^{0} du\, W'(|u|;z)\right.$$

$$\left.[2a_{zz}^{cl}(z,z+u)+a_{zz}^{cl}(z,z-u)]/[\pi k_o^2 K_o^*(z)]\right\}$$

Here the functions F_1, F_2, and F_3 are defined in Appendix D.

All of the functions in (46)-(48) are smooth enough so that no numerical difficulties should result in the solution of (46).

In the same way, the last equation in the set (34) becomes

$$E_{13} + n<EE^*>_{13} = \int_{\infty}^{0} dz_1 \left\{ N_{13}<EE^*>_{13} + 1/2\, g_{\rho zz} <EE^*>^*_{13} \right. \tag{49}$$

where

$$N_{13} = 1/2[g_{\rho\rho zz} + g_{\phi\phi zz}] \tag{50}$$

and the subscripted g's are identical to the c's in (38) and (39) except that the upper limit in the ρ integration is replaced by ρ_1 and a_{zz} is replaced by a_{zz}^{cl}. The multiplying factor n in (49) is

$$n = 7<|\xi|^2>/(8k_o^2)\left\{-F_2(\ell\rho_1)/[k_o^2|K_o(z)|^2]+\ell^2F_1(\ell\rho_1)/K_o^*(z)\right\} \tag{51}$$

$$+[4\pi k_o^2 K_o^*(z)]^{-1}\int_0^{\rho_1}\rho d\rho\, du\, W'(|u|;z)\Big\{2[a_{\rho\rho}(z,z+u)+a_{\phi\phi}(z,z+u)]$$

$$+a_{\rho\rho}(z,z-u)+a_{\phi\phi}(z,z-u)\Big\}$$

The integral equations (46) and (49) determine the second moment dyad $\underset{\approx}{E}$. It is interesting to observe that (49) is not coupled to (46) so that the off-diagonal components of $\underset{\approx}{E}$ may be determined independently of its diagonal elements. Also, notice that if the incident wave polarization is $\hat{h}_o$ (see eq (31), then $\underset{\sim}{E}^m$ in (28), will have only an $\hat{h}_o$ component [14]. Thus, the driving term in (49) will vanish and the off-diagonal elements of $\underset{\approx}{E}$ will be zero when the incident polarization is horizontal. On the other hand, all three diagonal elements of $\underset{\approx}{E}$ will, in general, be non-zero when (46) is solved for either horizontally or vertically polarized incident waves.

IV. CONCLUSIONS

A system of integral equations in one independent variable has been derived for the second moment dyad of the incoherent field. This system is well suited for numerical work because all singular behavior has been extracted from the integrands. The kernels in these equations are, at present, expressed as double integrals (see (47) and (50) together with (36-(39)). However, a preliminary examination of parts of these kernels has shown that one of the integrations (the integral over u) may be done analytically if the techniques discussed in [14] are followed. Thus, the numerical work will be simplified even further provided that the remaining parts of the kernels can be treated similarly.

The theory developed here has shown that a calculationally practical scheme for extending strong fluctuation theory beyond the distorted Born approximation for scattering problems is feasible.

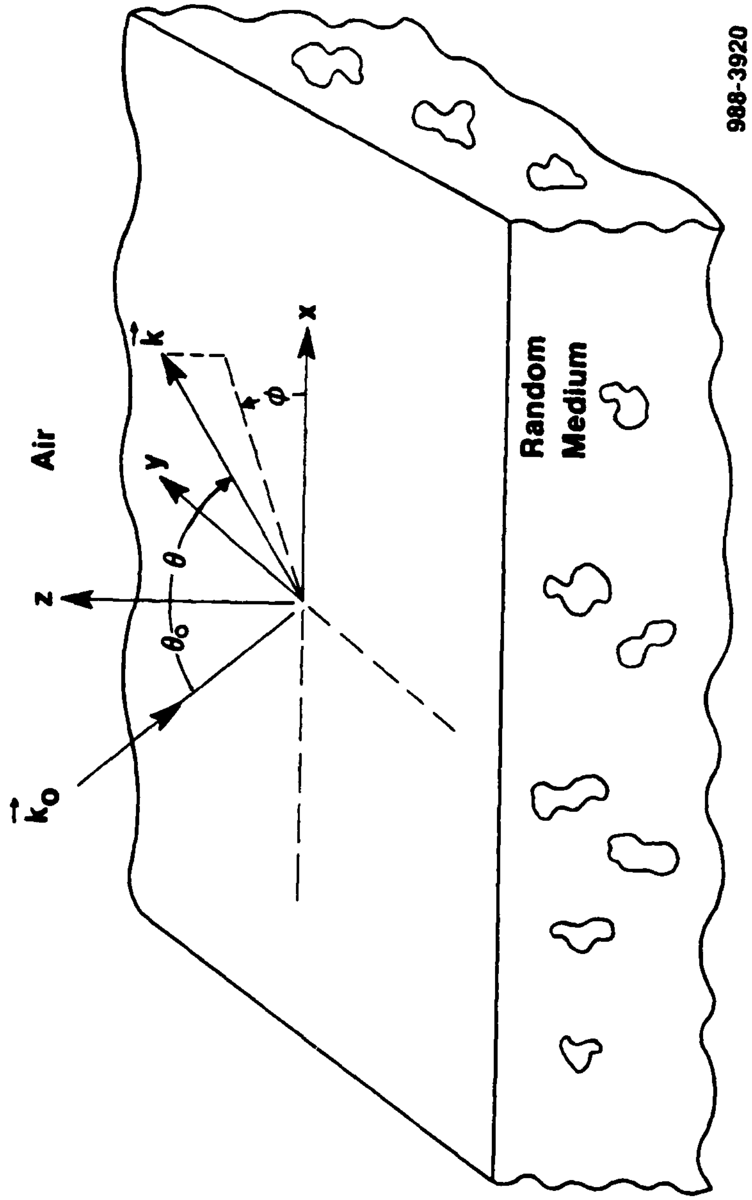

Figure 1 Scattering Geometry

APPENDIX A. GREEN'S FUNCTIONS

In Section II, it was shown that the Green's dyad $\underset{\approx}{\Gamma}$ for the operator (7) was required in order to determine the second moment of the incoherent field within the random medium. As in [14], it proves to be useful to represent $\underset{\approx}{\Gamma}$ as a two dimensional Fourier integral. The discussion in [14] shows that

$$\underset{\approx}{\Gamma}(\underset{\sim}{r},\underset{\sim}{r}') = (2\pi)^{-2}\int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \; \underset{\approx}{A}'(z,z',\xi,\eta)\exp\left\{i[\xi(x-x')+\eta(y-y')]\right\} \tag{A-1}$$

where

$$\underset{\approx}{A}' = \begin{pmatrix} a_{\phi\phi}+(\xi^2/\rho^2)(a_{\rho\rho}-a_{\phi\phi}) & (\xi\eta/\rho^2)(a_{\rho\rho}-a_{\phi\phi}) & (\xi/\rho)a_{\rho z} \\ (\xi\eta/\rho^2)\,(a_{\rho\rho}-a_{\phi\phi}) & a_{\phi\phi}+(\eta^2/\rho^2)(a_{\rho\rho}-a_{\phi\phi}) & (\eta/\rho)a_{\rho z} \\ (\xi/\rho)a_{z\rho} & (\eta/\rho)a_{z\rho} & a_{zz} \end{pmatrix} \tag{A-2}$$

Here $\rho^2 = \xi^2 + \eta^2$ and $a_{\rho\rho}$, $a_{\phi\phi}$, etc. are functions of z, z′ , and ρ. Substitution of (A1) and (A2) into $L_o\underset{\approx}{\Gamma} = \delta(\underset{\sim}{r}-\underset{\sim}{r}')$ shows, after some manipulation, that the following ordinary differential equations are satisfied by the components of the Green's dyad in the region $z<0$:

$$\frac{d^2 a_{\phi\phi}}{dz^2} + a\, a_{\phi\phi} = -\,\delta(z-z') \tag{A-3}$$

$$\frac{d^2 a_{\rho\rho}}{dz^2} - b\,\frac{da_{\rho\rho}}{dz} + a\, a_{\rho\rho} = -\left[a/(k_o^2 K_o)\right]\delta(z-z') \tag{A-4}$$

$$\frac{d^2 a_{\rho z}}{dz^2} - b\frac{da_{\rho z}}{dz} + a\, a_{\rho z} = -[i\rho/(k_o^2 K_o)]\frac{d\,\delta(z-z')}{dz} + [ib/\rho]\delta(z-z') \tag{A-5}$$

$$a_{z\rho} = [i\rho/a]\frac{da_{\rho\rho}}{dz} \tag{A-6}$$

$$a_{zz} = [i\rho/a]\frac{da_{\rho z}}{dz} - \delta(z-z')/a \tag{A-7}$$

where

$$a(z) = k_o^2 K_o(z) - \rho^2 \tag{A-8}$$

$$b(z) = \rho^2 \frac{dK_o}{dz}/(aK_o) \tag{A-9}$$

Continuity of the tangential components of the electric and magnetic fields at each point of discontinuity of K_o (say, at z=-d) requires that the components of the Green's dyad satisfy the conditions.

$$a_{\phi\phi}\Big|_{-d-o}^{-d+o} = 0 \tag{A-10a}$$

$$\frac{da_{\phi\phi}}{dz}\Big|_{-d-o}^{-d+o} = 0 \tag{A-10b}$$

$$a_{\rho\rho}\Big|_{-d-o}^{-d+o} = 0 \tag{A-11a}$$

$$(K_o/a)\frac{da_{\rho\rho}}{dz}\Big|_{-d-o}^{-d+o} = 0 \tag{A-11b}$$

$$a_\rho \Big|_{-d-o}^{-d+o} = 0 \tag{A12a}$$

$$(K_o/a)\ \frac{da_{\rho z}}{dz}\Big|_{-d-o}^{-d+o} = 0 \tag{A-12b}$$

These continuity conditions also hold at $z=0$ where $K_o(+0)=1$. These represent the boundary conditions at $z=0$. There is also a condition that the components of the Green's dyad behave as outgoing waves as $z \longrightarrow -\infty$.

It was shown in [3] that it is very convenient to work with the Riccati equation equivalents of (A-3)-(A-7) if the additional assumption (which is not a practical restriction) is made that K_o approaches a constant as $z \longrightarrow -\infty$. In this case, one may define functions $\alpha_\pm(z)$ by the equations

$$\frac{da_{\phi\phi}}{dz} = \alpha_\pm a_{\phi\phi} \tag{A-13}$$

where α_+ is used for $z>z'$ and α_- is closed for $z<z'$. The boundary condition at $z'=0$ and the outgoing wave condition as $z \longrightarrow -\infty$ imply

$$\alpha_+(0) = i\sqrt{k_o^2 - \rho^2} \tag{A-14}$$

$$\alpha_-(-\infty) = -i\sqrt{a(-\infty)} \tag{A-15}$$

Substituting (A-13) into (A-3) (for $z\neq z'$) shows that

$$\frac{d\alpha_\pm}{dz} + \alpha_\pm^2 + a = 0 \tag{A-16}$$

where (from A-10b)

$$\alpha_{\pm}\Big|_{-d-o}^{-d+o} = 0 \tag{A-17}$$

at a point of discontinuity of the dielectric constant at z= -d. Integrating (A-13) and noting the delta function in (A-3) shows that

$$a_{\phi\phi} = \begin{cases} \exp\left[\int_{z'}^{z} \alpha_{+}(z'')dz''\right] \Big/ \left[\alpha_{-}(z')-\alpha_{+}(z')\right] & \text{if } z>z' \\ \exp\left[-\int_{z}^{z'} \alpha_{-}(z'')dz''\right] \Big/ \left[\alpha_{-}(z')-\alpha_{+}(z')\right] & \text{if } z<z' \end{cases} \tag{A-18}$$

For some purposes, it is convenient to transform (A-18) by means of the reciprocity relation $\underset{\approx}{\Gamma}(\underset{\sim}{r}\ \underset{\sim}{r}')=\underset{\approx}{\Gamma}^{T}(\underset{\sim}{r}',\ \underset{\sim}{r})$ (where ^{T} means transpose). This implies that an alternate expression for $a_{\phi\phi}$ is

$$a_{\phi\phi} = \begin{cases} \exp\left[-\int_{z'}^{z} \alpha_{-}(z'')dz''\right] \Big/ \left[\alpha_{-}(z)-\alpha_{+}(z)\right] & \text{if } z>z' \\ \exp\left[\int_{z}^{z'} \alpha_{+}(z'')dz''\right] \Big/ \left[\alpha_{-}(z)-\alpha_{+}(z)\right] & \text{if } z<z' \end{cases} \tag{A-19}$$

In a similar fashion, functions $\beta_{\pm}(z)$ may be introduced by the equations

$$\frac{da_{\rho\rho}}{dz} = \beta_{\pm}\, a_{\rho\rho} \tag{A-20}$$

These satisfy

$$\frac{d\beta\pm}{dz} + \beta_\pm^2 - b\beta_\pm + a = 0 \tag{A-21}$$

where

$$\beta_+(0) = ia(0)/\left[K_o(0)\sqrt{k_o^2-\rho^2}\right] \tag{A-22}$$

$$\beta_-(-\infty) = -i\sqrt{a(-\infty)} \tag{A-23}$$

The continuity conditions for $a_{\rho\rho}$ lead to the conditions

$$(K_o/a)\beta\pm\Big|_{-d-o}^{-d+o} = 0 \tag{A-24}$$

for $\beta_\pm$ at a point of discontinuity of the dielectric constant. In terms of $\beta_\pm$, $a_{\rho\rho}$ is given by

$$a_{\rho\rho} = \begin{cases} a(z')\left[k_\sigma^2 K_o(z')\right]^{-1} \exp\left[\int_{z'}^{z} \beta_+(z'')dz''\right]/\left[\beta_-(z')-\beta_+(z')\right] & \text{if } z>z' \\ a(z')\left[k_o^2 K_o(z')\right]^{-1} \exp\left[-\int_{z}^{z'} \beta_-(z'')dz''\right]/\left[\beta_-(z')-\beta_+(z')\right] & \text{if } z<z' \end{cases} \tag{A25}$$

As in the case for $a_{\phi\phi}$, the reciprocity relation leads to the alternate representation

$$a_{\rho\rho} = \begin{cases} a(z)\left[k_o^2 K_o(z)\right]^{-1}\exp\left[-\int_{z'}^{z} \beta_-(z'')dz''\right]/\left[\beta_-(z)-\beta_+(z)\right] & \text{if } z>z' \\ a(z)\left[k_o^2 K_o(z)\right]^{-1}\exp\left[\int_{z}^{z'} \beta_+(z'')dz''\right]/[\beta_+(z)-\beta_+(z)] & \text{if } z<z' \end{cases} \tag{A-26}$$

The component $a_{\rho z}$ of the Green's dyad satisfies the same equation as $a_{\rho\rho}$ for $z \neq z'$. Hence, the functions $\beta_\pm$ may be used to describe $a_{\rho\rho}$. However, the delta function and its derivative in (A-5) result in the equation

$$a_{\rho z} = \begin{cases} -\left[i\rho\beta_-(z')/a(z')\right] a_{\rho\rho} & \text{if } z>z' \\ -\left[i\rho\beta_+(z')/a(z')\right] a_{\rho\rho} & \text{if } z<z' \end{cases} \qquad \text{(A-27)}$$

Thus, $a_{\rho z}$ is discontinuous at $z=z'$.

Since $a_{\rho\rho}$ and $a_{\rho z}$ are described by the functions $\beta_\pm$, equations (A-6) and (A-7) show that $a_{z\rho}$ and a_{zz} are also described by $\beta_\pm$. Thus

$$a_{z\rho} = \begin{cases} \left[i\rho\beta_+(z)/a(z)\right] a_{\rho\rho} & \text{if } z>z' \\ \left[i\rho\beta_-(z)/a(z)\right] a_{\rho\rho} & \text{if } z<z' \end{cases} \qquad \text{(A-28)}$$

Like $a_{\rho z}$, this is also discontinuous at $z=z'$. Finally

$$a_{zz} = \begin{cases} \left[i\rho\beta_+(z)/a(z)\right] a_{\rho z} & \text{if } z>z' \\ \left[i\rho\beta_-(z)/a(z)\right] a_{\rho z} & \text{if } z<z' \end{cases} \qquad \text{(A-29)}$$

For problems where integrals arise over an interval containing the point $z=z'$, an additional delta function contribution must be added to (A-29). This arises from the explicit delta function shown in (A-7) as well as the fact, evident from (A-5), that $\frac{da_{\rho z}}{dz}$ has a delta function part. The total term to be added to (A-29) is $-\left[1/(k_o^2\, K_o)\right]\delta(z-z')$.

It is of some interest to note the asymptotic behavior of the components of the Green's dyad as the parameter $\rho \to \infty$. A knowledge of this behavior is required in connection with integrations over intervals containing the point $z=z'$. We are particularly concerned with the behavior of terms which will lead to non-zero results for the integrals discussed in Section III. Notice that (A-8) and (A-9) show that

$$a(z) \to -\rho^2 \tag{A-30}$$

$$b(z) \to -\frac{dK_o}{dz} / K_o \tag{A-31}$$

Considering $\alpha_\pm$ first, it is seen that the boundary conditions (A-14) and A-15) reduce to

$$\alpha_+(0) \to -\rho \tag{A-32}$$

$$\alpha_-(-\infty) \to \rho \tag{A-33}$$

Thus (A-16) yields

$$\alpha_\pm(z) \to \mp\rho \tag{A-34}$$

independent of z so that (A-14) yields

$$a_{\phi\phi} \to [1/(2\rho)]\exp[-\rho|z-z'|] \tag{A-35}$$

For $\beta_{\pm}$, (A-22) yields

$$\beta(-\infty) \to \rho \tag{A-36}$$

and (A-21) yields

$$\beta_{-}(z) \to \rho \tag{A-37}$$

On the other hand, (A-22) does not lead to $\beta_{+}(0) \to -\rho$. However, as a function of z, the solution of (A-21) rapidly approaches

$$\beta_{+}(z) \to -\rho \tag{A-38}$$

with the thickness of the transition zone approaching zero as $\rho \to \infty$. With these results, (A-29) yields

$$a_{\rho\rho} \to -[\rho/(2k_o^2 K_o(z'))] \exp[-\rho|z-z'|] \tag{A-39}$$

These results immediately imply

$$a_{\rho z} \to -[i\rho\, \mathrm{sgn}(z-z')/(2k_o^2 K_o(z'))] \exp[-\rho|z-z'|] \tag{A-40}$$

and

$$a_{z\rho} \to -[i\rho\, \mathrm{sgn}(z-z')/(2k_o^2 K_o(z'))] \exp[-\rho|z-z'|] \tag{A-41}$$

where sgn(z-z′) is the sign of z-z′. Since the asymptotic forms of $a_{\rho z}$ and $a_{z\rho}$ simply change sign at $z=z'$, integration in a small symmetrically placed interval containing this point will lead to zero. Of course, if $|z-z'|$ is not small, large values of ρ imply that integrals containing $a_{\rho z}$ and $a_{z\rho}$ will approach zero because of the exponential factors. Finally,

$$a_{zz} \rightarrow \left[\rho/(2\, k_o^2\, K_o(z'))\right] \exp\left[-\rho\, |z-z'|\right] - \delta(z-z')/(k_o^2\, K_o(z')) \tag{A-42}$$

APPENDIX B ANALYSIS OF EXPLICIT DELTA FUNCTION PARTS IN KERNEL

According to Appendix A, those parts of the integrals of eqs. (36) and (38) involving a_{zz} contain an explicit term with a delta function. In fact, Appendix A gives

$$a_{zz}(z,z_1) = a_{zz}^{cl} - \delta(z-z_1)/(k_o^2 K_o) \tag{B-1}$$

where a_{zz}^{cl} is a function in the classical sense containing no delta functions and is given by eq. (A-29).

Using (B-1) allows b_{zz}(eq (36)) to be written as

$$b_{zz} = (2\pi)^{-1} \, \mathrm{Re}\int_0^\infty \rho d\rho \int_{-\infty}^{0} du \; W'(|u|;z_1) \Big\{ a_{zz}^{cl}(z_1) a_{zz}^{cl*}(z_1+u)$$

$$-a_{zz}^{cl}(z_1)\delta(z-z_1-u)/[k_o^2 K_o^*(z)] - a_{zz}^{cl*}(z_1+u)\delta(z-z_1)/[k_o^2 K_o(z)]$$

$$+\delta(z-z_1)\delta(z-z_1-u)/\big[k_o^4 |K_o|^2\big]\Big\}$$

(B-2)

$$\approx b_{zz}^{cl} - (2\pi)^{-1} \mathrm{Re}\int_\infty^\infty \rho d\rho \Big\{ W'(|z-z_1|;z) a_{zz}^{cl}(z_1)/[k_o^2 K_o^*(z)] + \int_{-\infty}^{0} du \; W'(|u|;z)$$

$$a_{zz}^{cl*}(z+u)\delta(z-z_1)/[k_o^2 K_o] - \delta(z-z_1) W'(0;z)/[k_o^4 |K_o|^2]\Big\}$$

where b_{zz}^{cl} is given by (36) with a_{zz} replaced by a_{zz}^{cl} and, in the second equation of (B-2), the approximation $W'(|z-z_1|;z_1) \approx W'(|z-z_1|;z)$ is used. When (B-2) is used in the expression for E_{33}(see(34) , an intergration over z_1 is performed. We consider the contribution to this integral from the separated delta function parts of a_{zz}. Thus

$$I = 2\int_{-\infty}^{0} dz_1 [b_{zz}^{cl} - b_{zz}] \langle EE^* \rangle_{33}(z_1)$$

$$= \pi^{-1} \mathrm{Re} \int_0^{\infty} \rho d\rho \Big\{ \int_{-\infty}^{0} dz_1 W'(|z-z_1|;z_1) a_{zz}^{cl}(z_1) \langle EE^* \rangle_{33}(z_1) / [k_o^2 K_o^*(z)]$$
$$+ \int_{-\infty}^{0} du W'(|u|;z) a_{zz}^{cl}(z+u) \langle EE^* \rangle_{33}(z) / [k_o^2 K_o(z)]$$
$$- W'(0;z) \langle EE^* \rangle_{33}(z) / \big[k_o^4 |K_o(z)|^2\big] \Big\} \tag{B-3}$$

In the first term of (B-3), the argument of $\langle EE^* \rangle_{33}$ may be replaced by z because of the sharply peaked behavior of W'. Further, except for z within a few correlation lengths of the surface z=0, the upper limit of the first integral within the curly brackets may be replaced by ∞ and the variable of integration changed to $u = z-z_1$. Then, making use of the fact that $\langle EE^* \rangle_{33}$ is real,

$$I = d_{zz} \langle EE^* \rangle_{33}(z) \tag{B-4}$$

and

$$2\int_{-\infty}^{0} dz_1 b_{zz} \langle EE^* \rangle_{33}(z_1) = 2\int_{-\infty}^{0} dz_1 b_{zz}^{cl} \langle EE^* \rangle_{33}(z_1) - d_{zz} \langle EE^* \rangle_{33}(z) \tag{B-5}$$

where the multiplying factor d_{zz} is given by

$$d_{zz} = (\pi k_o^2)^{-1} \mathrm{Re} \int_0^{\infty} \rho d\rho \Big\{ \int_{-\infty}^{0} du\, W'(|u|;z) \big[2a_{zz}^{cl}(z+u) + a_{zz}^{cl}(z-u)\big] / K_o^*(z)$$
$$- W'(0;z) / \big[k_o^2 |K_o(z)|^2\big] \Big\} \tag{B-6}$$

A similar treatment of a_{zz} in (39) shows that the first term in the last equation of the set (34) may be written as

$$\int_{-\infty}^{0} dz_1 D_{13} \langle EE^* \rangle_{13}(z_1) = \int_{-\infty}^{0} dz_1 D_{13}^{cl} \langle EE^* \rangle_{13}(z_1) - d_{\rho\phi} \langle EE^* \rangle_{13}(z) \tag{B-7}$$

where D_{13}^{cl} is the same as D_{13} except that all terms containing a_{zz} are replaced by a_{zz}^{cl}.

The multiplying factor $d_{\rho\phi}$ in (B-7) is

$$d_{\rho\phi} = [4\pi k_o^{2} K_o^{*}(z)]^{-1} \int_0^{\infty} \rho d\rho \int_{-\infty}^{0} du\, W'(|u|;z) \qquad \text{(B-8)}$$

$$\left\{ 2\left[a_{\rho\rho}(z+u)+a_{\phi\phi}(z+u)\right]+ a_{\rho\rho}(z-u)+a_{\phi\phi}(z-u) \right\}$$

APPENDIX C INTEGRALS FOR LARGE PARAMETER ρ

When eqs (35) - (39) are substituted into (34) and use is made of (40), it is seen that integrals of the form

$$\int_{-\infty}^{0} dz_1 \int_{\rho_1}^{\infty} \rho d\rho \int_{-\infty}^{0} du = \int_{-\infty}^{z} dz_1 \int_{\rho_1}^{\infty} \rho d\rho \int_{-\infty}^{0} du + \int_{z}^{0} dz_1 \int_{\rho_1}^{\infty} \rho d\rho \int_{-\infty}^{0} du \qquad \text{(C-1)}$$

arise. The asymptotic forms for the compohents of the Green's dyad given in Appendix A may be used as an approximation for all of the a's shown in (36) - (39) if $\rho > \rho_1$. When this is done, all integrands contain a factor of the form

$$a(z, z_1+u) \propto \exp[-\rho |z-z_1-u|] \qquad \text{(C-2)}$$

where the subscripts on the a have been omitted and, in contrast to eqs (34)-(39), the first argument, z, has been included because of its role in the following analysis.

Consider the first term in (C-1) where $z > z_1$. Eq (C-2) and the results of Appendix A show that

$$a(z, z_1+u) = a(z, z_1)\exp(\rho u) \qquad \text{(C-3)}$$

provided that the approximation $K_o(z_1+u) \approx K_o(z_1)$ is used whenever necessary. For large ρ, $a(z, z_1)$ is a sharply peaked function of $|z-z_1|$. Thus, more slowly varying functions of z_1 may be removed from the integrand when performing integrals over z_1. Then, denoting any pair of a's that occur in the integrands by a_1 and a_2, we find

$$\int_{-\infty}^{z} dz_1 \int_{\rho_1}^{\infty} \rho d\rho \; a_1(z, z_1) a_2^*(z, z_1) F(\rho, z_1) \approx \int_{\rho_1}^{\infty} \rho d\rho \, F(\rho, z) \int_{-\infty}^{z} dz_1 a_1(z, z_1) a_2^*(z, z_1)$$

(C-4)

where $F(\rho,z_1)$ depends on the integral over u of W' $\exp(\rho u)$ and the components of $<\underset{\approx}{EE}^*>$

On the other hand, for $z<z_1$, $W'(|u|;z_1)$ has a sharp peak at u=0 while $\exp[-\rho|z-z_1-u|]$ has a sharp peak at $u= z-z_1$. Therefore, over most of the range (z,0) where $z_1\neq z$, we are dealing with the product of two sharply peaked functions whose peaks are separated. Hence, the contribution of the second term in (C-1) is negligible compared to the first.

Thus, the integrals in (34) for which $\rho>\rho_1$ may be approximated as

$$\int_{-\infty}^{0} dz_1 \int_{\rho_1}^{\infty} d\rho \int_{-\infty}^{0} du\ W'(|u|;z_1)a_1(z,z_1)a_2^*(z,z_1+u)<EE^*>(z_1)$$

$$\approx \int_{\rho_1}^{\infty}\rho d\rho\left[\int_{-\infty}^{0} du\ W'(|u|;z)e^{\rho u}\right]\left[\int_{-\infty}^{z} dz_1 a_1(z,z_1)a_2^*(z,z_1)\right]<EE^*>(z) \qquad \text{(C-5)}$$

In particular, using the asymptotic resutls in Appendix A, we find the following for integrals over the range $(-\infty,z)$:

$$\int_{-\infty}^{z} dz_1 a_{\phi\phi}(z,z_1)a_{\phi\phi}^*(z,z_1)\approx 1/\left[8\rho^3\right]$$

$$\int_{-\infty}^{z} dz_1 a_{\rho\rho}(z,z_1)a_{\rho\rho}^*(z,z_1)\approx\int_{-\infty}^{z} dz_1 a_{zz}^{cl}(z,z_1)a_{zz}^{cl*}(z,z_1)\approx\rho/\left[8k_o^4|K_o(z)|^2\right]$$

$$\int_{-\infty}^{z} dz_1 a_{\rho\rho}(z,z_1)a_{zz}^{cl*}(z,z_1)\approx -\rho/\left[8k_o^4|K_o(z)|^2\right] \qquad \text{(C-6)}$$

$$\int_{-\infty}^{z} dz_1 a_{\rho\rho}(z,z_1)a_{zz}^{cl*}(z,z_1)\approx -1/\left[8k_o^2K_o^*(z)\rho\right]$$

$$\int_{-\infty}^{z} dz_1 a_{\phi\phi}(z,z_1)a_{zz}^{cl*}(z,z_1)\approx 1/\left[8k_o^2K_o^*(z)\rho\right]$$

$$\int_{-\infty}^{z} dz_1 a_{\rho z}(z,z_1)a_{\rho z}^*(z,z_1)\approx\int_{-\infty}^{z} dz_1 a_{z\rho}(z,z_1)a_{z\rho}^*(z,z_1)\approx 0$$

In (C-6), a_{zz}^{cl} is the part of a_{zz} which does not contain an explicit delta function (see Appendix B).

APPENDIX D EXPLICIT INTEGRALS FOR EXPONENTIAL CORRELATION FUNCTION

In this appendix, the completion of the evaluation of the integrals shown in (C-5) will be made assuming that the function W' is given by (43) and (44). The integral over u in (C-5) is elementary and results in

$$\int_{-\infty}^{0} du\, W'\, e^{\rho u} = \left[2\pi \langle |\xi|^2 \rangle / \ell\right] p^{-3} \left\{ [p+\rho]^{-1} + p[p+\rho]^{-2} \right\} \tag{D-1}$$

Substituting (D-1) in (C-5) and noting that (C-6) will introduce various powers of ρ according to which a's occur, we find that three kinds of integrals arise:

$$\int_{\rho_1}^{\infty} d\rho \begin{pmatrix} 1 \\ \rho^2 \\ \rho^{-2} \end{pmatrix} p^{-3} \left\{ [p+\rho]^{-1} + p[p+\rho]^{-2} \right\} = \begin{pmatrix} \ell^3 F_1(\ell\rho) \\ \ell F_2(\ell\rho) \\ \ell^5 F_3(\ell\rho) \end{pmatrix} \tag{D-2}$$

where, upon using the substitution $x = \ell\rho$ in the left hand side of (D-2), it is found that

$$F_{\begin{pmatrix}1\\2\\3\end{pmatrix}}(X) = \int_X^{\infty} dx \begin{pmatrix} 1 \\ x^2 \\ x^{-2} \end{pmatrix} [1+x^2]^{-3/2} \left[(1+x^2)^{1/2} + x\right]^{-1} \left\{ 1 + (1+x^2)^{1/2} \left[(1+x^2)^{1/2} + x\right]^{-1} \right\} \tag{D-3}$$

Although, it does not appear possible to express the integrals in (D-3) exactly in terms of elementary functions, numerically satisfactory approximations are not difficult to obtain. First, we consider limiting behavior. For large X, all of the roots in (D-3) may be expanded in powers of $1/x^2$. This leads to

$$F_1(X) = 1/(4X^3) - 1/(4X^5) + \ldots$$

$$F_2(X) = 3/(4X) - 5/(12X^3) + \ldots \qquad \text{(D-4)}$$

$$F_3(X) = 3/(20X^3) - 5/(28X^7) + \ldots$$

For small X, it is found that

$$F_1(X) = 1 - 2X + \frac{3}{2}X^2 + \ldots$$

$$F_2(X) = \frac{2}{3} - \frac{2}{3}X^3 + \frac{3}{4}X^4 + \ldots \qquad \text{(D-5)}$$

$$F_3(X) = 2/X + 3\ \ell n\ X + \ldots$$

Using (D-4) and (D-5) as guides for developing functional forms, a numerical calculation of a rational Chelbychev fit yielded

$$F_1(X) = (.83448 - .71644X + .21851\ X^2) / (.83473 + .93764X + X^2)$$

$$F_2(X) = \frac{2}{3} - X^3\ (.87871 - .58270X + .15574X^2) / (1.31823 + .59901X + X^2)$$

$$F_3(X) = 2/X + 3\ell nX - (3.1544 + .6467X + 5.0162X^2) / (2.9216 + .6307X + X^2)$$

(D-6)

for X < 1 while

$$F_1(X) = (.25003\ X^2 - 1.375 \times 10^{-2}) / \left[X^3 (X+^2 .94783)\right]$$

$$F_2(X) = (.74972\ X^2 + .13296) / \left[X\ (X^2 + .72405)\right] \qquad \text{(D-7)}$$

$$D_3(X) = (.15010\ X^2 - .01927) / \left[X^5\ (X^2 + 1.07866)\right]$$

for $X \geq 1$.

Comparison of numerical results from (D-6) and (D-7) with high precision numerical evaluation of (D-3) showed that the rational approximations to F_1, F_2, F_3 have maximum errors of .06%, .03%, and .06%, respectively.

The integrals in (C-5) are completely specified by F_1, F_2, F_3 and the various numerical factors shown in (D-1), (D-2), and (C-6).

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